

Spatially structured oscillations in a two-dimensional excitatory neuronal network with synaptic depression

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Abstract

Excitatory neuronal networks with synaptic depression can exhibit spatially structured oscillations. We study the dynamics of a two-dimensional excitatory neuronal network with synaptic depression. The network is modeled by a set of coupled differential equations. We show that the network exhibits spatially structured oscillations in the form of traveling waves. The waves propagate through the network and their speed depends on the strength of the synaptic depression. We also show that the network exhibits a transition from a steady state to a state with spatially structured oscillations as the strength of the synaptic depression is increased. This transition is characterized by a bifurcation diagram. The bifurcation diagram shows that the network undergoes a Hopf bifurcation as the strength of the synaptic depression is increased. The Hopf bifurcation is characterized by the appearance of a pair of complex conjugate eigenvalues with a positive real part. The Hopf bifurcation is also characterized by the appearance of a limit cycle in the bifurcation diagram. The limit cycle represents a state with spatially structured oscillations. We also show that the network exhibits a transition from a steady state to a state with spatially structured oscillations as the strength of the synaptic depression is increased. This transition is characterized by a bifurcation diagram. The bifurcation diagram shows that the network undergoes a Hopf bifurcation as the strength of the synaptic depression is increased. The Hopf bifurcation is characterized by the appearance of a pair of complex conjugate eigenvalues with a positive real part. The Hopf bifurcation is also characterized by the appearance of a limit cycle in the bifurcation diagram. The limit cycle represents a state with spatially structured oscillations.

Keywords

Spatially structured oscillations, synaptic depression, traveling waves, bifurcation diagram, Hopf bifurcation, limit cycle

1 Introduction

Spatially structured oscillations in a two-dimensional excitatory neuronal network with synaptic depression. We study the dynamics of a two-dimensional excitatory neuronal network with synaptic depression. The network is modeled by a set of coupled differential equations. We show that the network exhibits spatially structured oscillations in the form of traveling waves. The waves propagate through the network and their speed depends on the strength of the synaptic depression. We also show that the network exhibits a transition from a steady state to a state with spatially structured oscillations as the strength of the synaptic depression is increased. This transition is characterized by a bifurcation diagram. The bifurcation diagram shows that the network undergoes a Hopf bifurcation as the strength of the synaptic depression is increased. The Hopf bifurcation is characterized by the appearance of a pair of complex conjugate eigenvalues with a positive real part. The Hopf bifurcation is also characterized by the appearance of a limit cycle in the bifurcation diagram. The limit cycle represents a state with spatially structured oscillations. We also show that the network exhibits a transition from a steady state to a state with spatially structured oscillations as the strength of the synaptic depression is increased. This transition is characterized by a bifurcation diagram. The bifurcation diagram shows that the network undergoes a Hopf bifurcation as the strength of the synaptic depression is increased. The Hopf bifurcation is characterized by the appearance of a pair of complex conjugate eigenvalues with a positive real part. The Hopf bifurcation is also characterized by the appearance of a limit cycle in the bifurcation diagram. The limit cycle represents a state with spatially structured oscillations.

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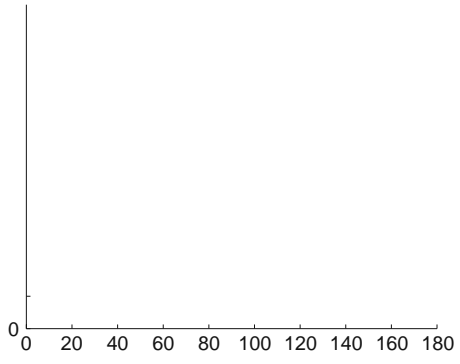
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$$f_u = H_u - \dots = \dots \quad (.)$$

$$1, r_t, t, \dots ((S, r_t, \dots) (1 - \dots) T / C_i \dots 001, \dots 1 \quad 0T \quad 0T \quad (1)T / C_i \dots 0.0 \quad 1.0 \quad 1.0$$

$$\left(\begin{array}{c} \mathbf{u} \\ \mathbf{v} \end{array} \right) + \left(\begin{array}{c} \mathbf{r} \\ \mathbf{f} \end{array} \right) = \left(\begin{array}{c} \mathbf{t} \\ \mathbf{t} \end{array} \right) \quad (1)$$



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- A

$$L_h \frac{u_{ij}^{k+} - u_{ij}^k}{t} + u_{ij}^{k+} = M q_{ij} f, u_{ij} \quad (.1)$$

$$L_h \frac{q_{ij}^{k+} - q_{ij}^k}{t} = - q_{ij} f, u_{ij} \quad (.)$$

$i = 1, \dots, N_x, j = 1, \dots, N_y, L_h \neq t$
 $u_{ij} = q_{ij} f, u_{ij} \quad (.1),$
 $u_{ij} = q_{ij} f, u_{ij} \quad (.1),$

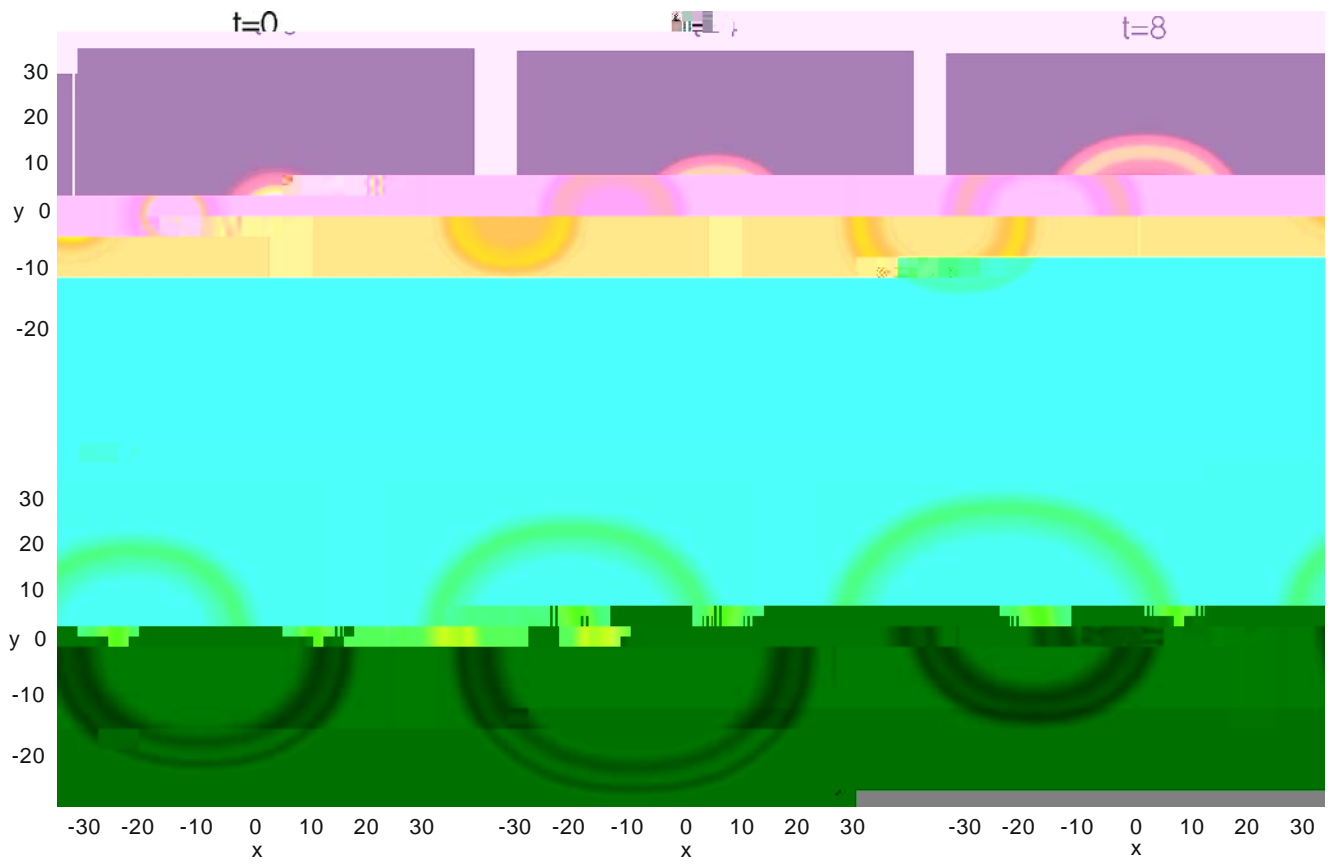


Fig. 13 S_i at $t=0, 1, 8$ and $u(x, y)$.

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$$a \dots \frac{a}{s} J, \dots J, \dots \dots = \frac{a}{s} I, \dots sa K, \dots sa.$$

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$$\dots + \dots = \dots a. \dots (.1)$$

t

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$$= - a I, a K, a - \frac{a}{s} I, a K, a \dots (.1)$$

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