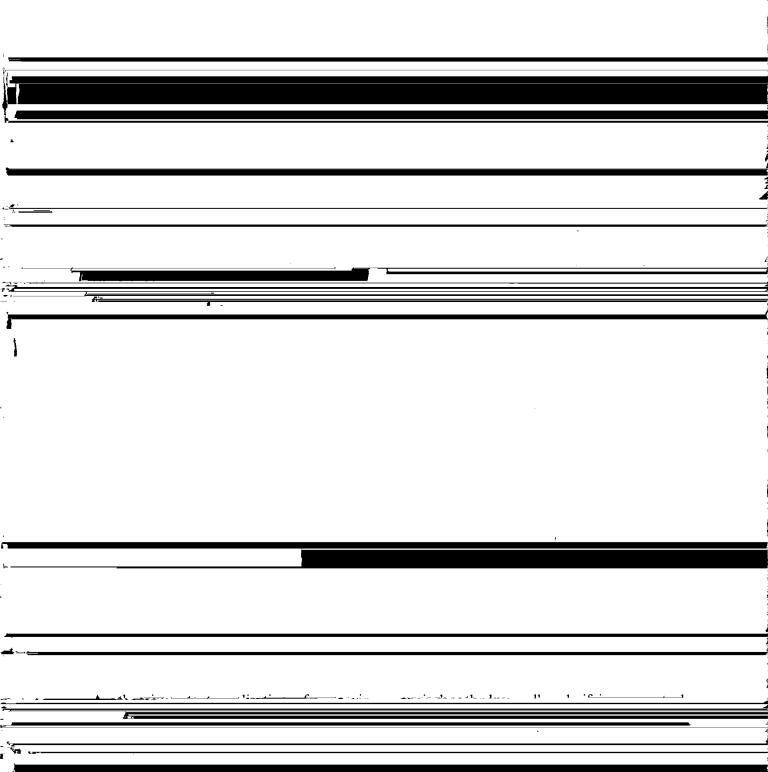
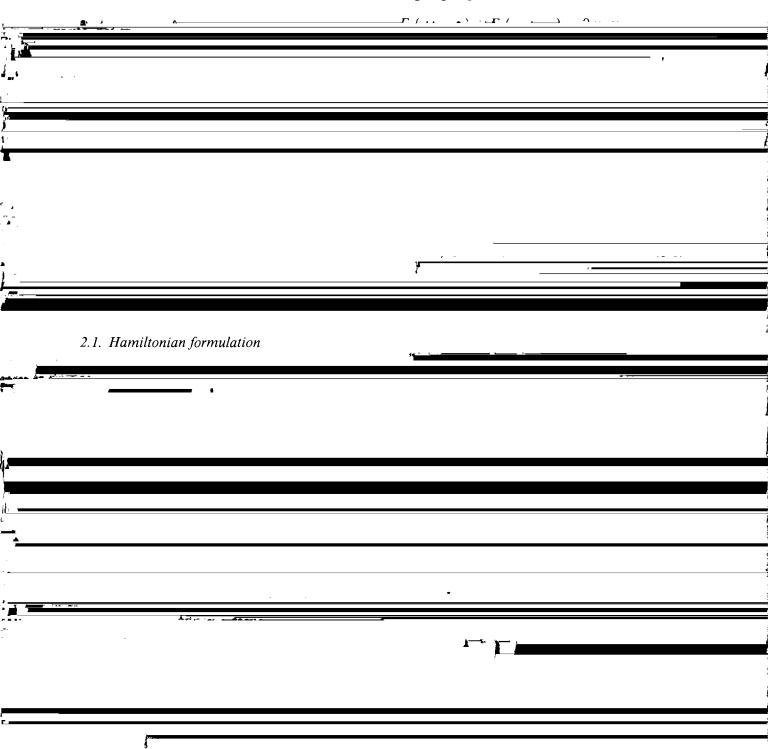
ter the field somewhat, but for moderate plasma pressures the magnetic field lines will tend to remain close to nested tori. Because the magnetic field causes plasma properties to be highly anisotropic, it is desirable to use a curvilinear coordinate system, with level surfaces of one of the coordinates corresponding to the approximately interiors.

type for Hamiltonian systems more susceptible to numerical experimentation than continuous time systems, and are consequently already well explored, especially the standard map (see e.g. MacKay et al. [4]). Since these maps are much simpler than full Hamiltonian systems (especially those corresponding to magnetic field lines in plasma confinement problems), their



strained variations in appendix C.

is stationary for all variations of  $(x_0, x_1, \dots, x_n)$  with  $x_0$  and  $x_n$  held fixed. This yields the Euler–Lagrange equations



in the Lagrangian description before translating them into the Hamiltonian represenconstant term in eq. (2.10) or eq. (2.11) analogous to K in eq. (2.7).

parity-reversal (P) symmetries of a dynami- the condition that

cal system as the properties that, given any orbit segment  $(x_0, x_1, \dots, x_n)$ , the sequences  $(x_n, x_{n-1}, \dots, x_0)$  and  $(-x_0, -x_1, \dots, -x_n)$ , respectively, are also orbit segments. In terms of

$$F(x, x^*) = F(-x^*, -x) + R(x) - R(x^*),$$
(2.12)

## 2.6. Examples

As an example, consider the generalized standard map

where k is the nonlinearity parameter. This is an even function so the map

$$x^* = x + y - \frac{k}{2\pi} \sin 2\pi x,$$

This satisfies	eq.	(2.10)	with	Q(x)	=	-V	(x)	,

enperated hut in Township Earthantandered

Theorem 1. True intersections of  $\varphi_2$ -extremizing rotational curves C and  $C^*$  generated by an invertible circle map  $\rho$  belong to families which are orbits under the area-preserving map T.

To see this, let there be a true intersection at  $\theta = \theta_0$ . That is, let  $\Delta Y(\theta_0) = 0$ . Then the

(6.7) and  $x_n \equiv x_0 + m$ . Then the first variation of the action

$$W_{m,n} \equiv \sum_{j=0}^{n-1} F(x_j, x_{j+1})$$
 (6.8)

is zero because  $\Delta Y(\theta_i)$  is zero. Calculating the

rotation we have full control over their rotation numbers. It is these solutions which would apngr-19 provide the besis for defining a concret ized action-angle representation. One could use a truncated Farey tree construction to define the principal resonances in the domain of interest van-In

tween each resonance) and use the curves  $C, C^*$ , or the time-symmetric curve specified parametrically by  $x = X(\theta), y = \frac{1}{2}[Y_{+}(\theta) + Y_{-}(\theta)]$ to define a basic ladder of new momentum co-

ciently small k. The transformation to the new phase-space coordinates would then be com-

ing to all irrational rotation numbers since these are not in general smooth and are not continuously connected to the resonance surfaces).

We have studied only the lowest order reso-

a rotational invariant curve or a cantorus. In the former case  $\varphi_2$  is obviously a local (and global)

tions as the control parameter is varied would also be interesting to investigate, as well as the implications of this method for the theory of transport in area preserving maps.

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## Appendix A. Circle map identity

used to prove relationships (sum rules) between the Fourier coefficients of a circle map, its sumdifference representation and its inverse. We shall work in x-space, though similar relations

$$\int_{-1}^{1} F(x^* - x) \left[ x'_{\perp}(\eta) - x'(\eta) \right] d\eta \equiv 0, \quad (A.1)$$

for any integrable function F(x) = f'(x). Here  $x^* - x$  is a shorthand for  $x_+(\eta) - x_-(\eta)$ . Equation (A.1) follows by recognizing that the integrand is the perfect differential  $df(x^*-x)$  and

$$\int_{-\infty}^{1} F(x^* - x) x'(n) \, \mathrm{d}n$$

$$\equiv \frac{1}{2} \int_{0}^{1} F(x^* - x) [x'_{+}(\eta) + x'_{-}(\eta)]. \quad (A.2)$$

In particular, choosing  $F(\cdot) \equiv \cdot$  and  $\eta =$ 

sentation of  $\alpha$  corresponding to eq. (3.2) is simply  $-\Omega$ .

## Appendix B. Time-symmetric representation

A representation in which a reversionity otherwise) of the map  $\rho:\theta\mapsto\theta^*$  is manifest is