



# Normative decision rules in changing environments

Nicholas W Barendregt<sup>1\*</sup>, Joshua I Gold<sup>2</sup>, Krešimir Josi<sup>3</sup>, Zachary P Kilpatrick<sup>1</sup>

<sup>1</sup>Department of Applied Mathematics, University of Colorado Boulder, Boulder, United States; <sup>2</sup>Department of Neuroscience, University of Pennsylvania, Philadelphia, United States; <sup>3</sup>Department of Mathematics, University of Houston, Houston, United States

**Abstract** We study normative decision rules in a laboratory task that changes over time. We find that subjects do not follow the normative rule, but instead follow a rule that is more consistent with the initial conditions. We show that this behavior can be explained by a simple model of decision making that takes into account the changing environment. We also show that subjects are more likely to follow the normative rule when the environment is stable. Our results suggest that subjects use a simple heuristic to make decisions in changing environments.

\*For correspondence: nicholas.barendregt@colorado.edu

Competing interest: 19

Funding: 19  
Preprinted: 29 | 2022  
Received: 03 | 2022  
Accepted: 20 | 2022  
Published: 25 | 2022

## Editor's evaluation

The manuscript is well written and clearly presents the research question and findings. The authors provide a thorough discussion of the results and their implications. The manuscript is a high-quality contribution to the field of decision making in changing environments.

## Introduction

Normative decision rules are those that maximize expected utility. In a changing environment, the normative rule is not constant. We study how subjects behave in a laboratory task that changes over time. We find that subjects do not follow the normative rule, but instead follow a rule that is more consistent with the initial conditions. We show that this behavior can be explained by a simple model of decision making that takes into account the changing environment. We also show that subjects are more likely to follow the normative rule when the environment is stable. Our results suggest that subjects use a simple heuristic to make decisions in changing environments.





(n)nal)ca... an n all... n nm nalt a, s {s+,s-}, a n l... mnt c... al na... n nm nalt a, ... m d m n t, xi, a... a... f±(xi) = f(xi|s±) a... m n... c... c... al n... m n... f±. n al a... an... 45... n... a... xi:n a a n... n n a... m, ... - m ... l y\_n... n...

$$y_n = \ln \frac{\Pr(s_+ | \xi_{1:n})}{\Pr(s_- | \xi_{1:n})} = \ln \frac{f_+(\xi_n)}{f_-(\xi_n)} + y_{n-1}. \tag{2}$$

n... n... a... c... m... c... c... m an... n... a... m... (m d m n t)... a... m... a... a... l... a... c... c... n, ... a... c... m m... a... c... m... a... a... l... V+... c... s+, a al V\_-... c... s-, a n a al V\_w... a m l... a n), c a r... c... al... n... c... n...

$$V(p_n; \rho) = \max\{V_+(p_n; \rho), V_-(p_n; \rho), V_w(p_n; \rho)\}$$

$$= \max$$

m l l a c a g t t a g l c t o n l a c o m l t t o l n a m t a t m m a -  
 n m t t a l t o l c a g t m o t t m o t t c o n t o n t a f a n t  
 o m t m l m o n o c a g t t o n a a m t ( ).  
 o n t a n a g o t t l t o l m o t t t o n t o n m t t t g l  
 c a g t n a t a a m t t a e a g t t t t m t  $R_i = 0$  a n t t m  
 a  $R_c$  a g t a l t o a t n a m t a t t a a t t o n

$$R_c(t) = (R_2 - R_1)H_{\theta}(t - 0.5) + R_1. \tag{5}$$

t t a t c t o m e a g a  $R_1$  o t e a g a  $R_2$  a  $t = 0.5$ .  
 o t t g l e a g t t t o m a t t o l n a m t t t a l m o t a n t o m  
 c a t t t m l t o c o l l a t t t o l c a a t c o t t c t o n m o t t o  
 c a t t t o l n a m t t c c a l l c a a t t o n t n a m t m o t t t o n t  
 o t t l t c c a g t t a c o n t t t o n c o m n a o t t o - a n t -  
 c a g t a a l t (



... e ag d'v, mal' a g t' n nm n' m l' c ag t' n' u  
 al' a c a a' t' ol' nanc' a a a' c ag t' n a a' mla' m'  
 a a' c ag t' n' a' ( ). t' a g' m' l' l'  
 t' ol' m' t', g' anc, n' n nm n' t' g' l' c ag t' n' u' al'  
 $m = \frac{2^2}{2}$  a' g' a' a' t' n'

$$\mu(t) = (\mu_2 - \mu_1)H_{\theta}(t - 0.5) + \mu_1. \quad ( )$$

... t' g' l' e ag d'v, g' a n' n' t' mla' t' ol' m' t' m' t' n' a e ag  
 d'v' ( ). m' , n' t' a' m' m' c ag t' n' u' al' a' t' m' c  
 m' m' c ag t' n' t' n' a' . t' t' a' n' m' l' a' p' -41.994 a' p' -41.993 - . 5/

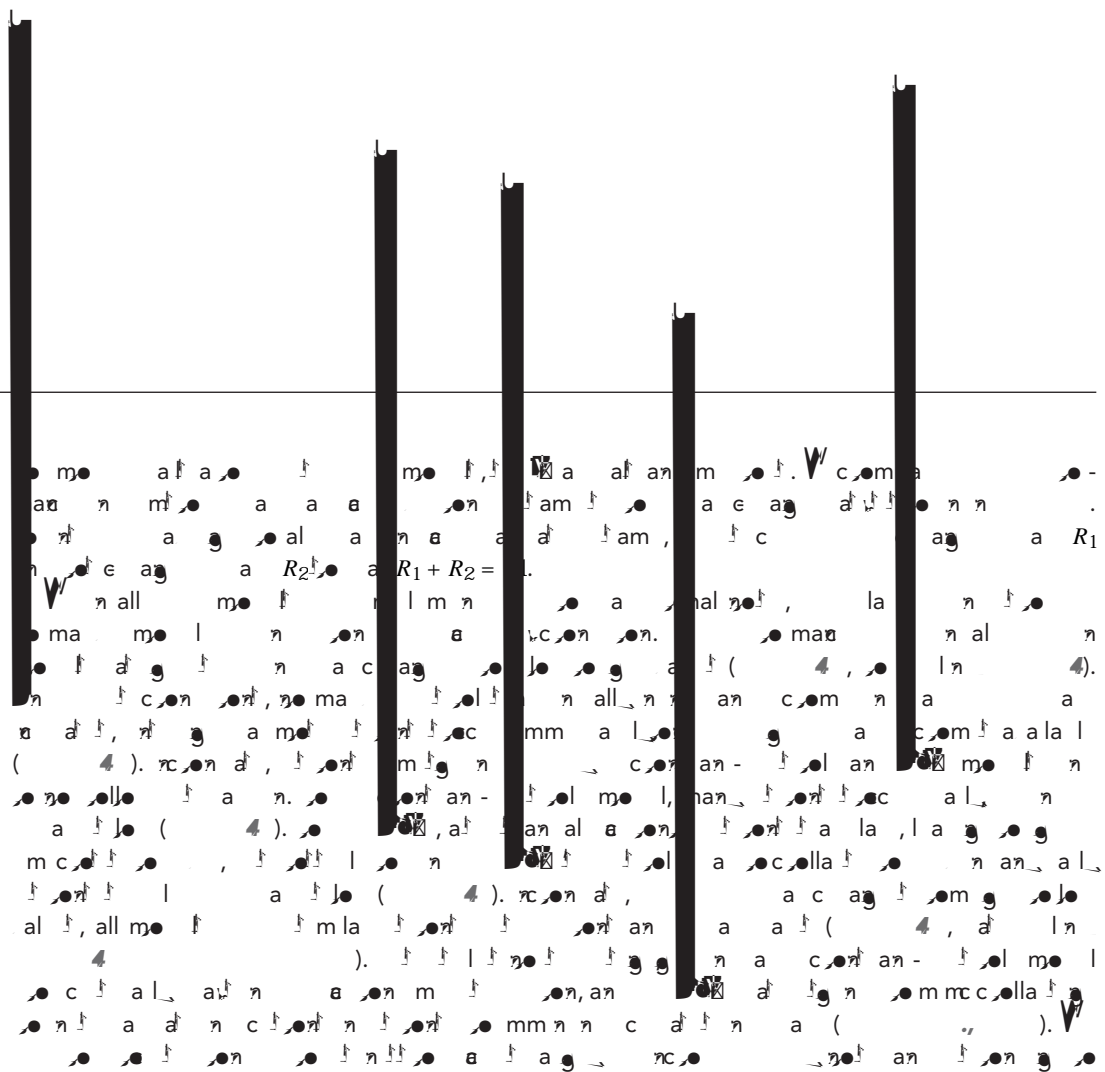
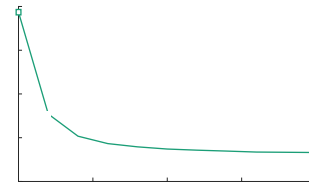




Figure 1. A schematic diagram illustrating the model structure. The diagram shows a network of nodes and connections, with a central node labeled 'm' and several peripheral nodes labeled 'a', 'b', 'c', 'd', 'e', 'f', 'g', 'h', 'i', 'j', 'k', 'l', 'm', 'n', 'o', 'p', 'q', 'r', 's', 't', 'u', 'v', 'w', 'x', 'y', 'z'. The connections are represented by lines of varying thickness, indicating different strengths or types of interactions. The diagram is divided into several regions, with a central region containing the most complex network of connections. The overall structure suggests a hierarchical or modular organization of the system.





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$$\begin{aligned} V(p_n; \rho) &= \max\{V_+(p_n; \rho), V_-(p_n; \rho), V_w(p_n; \rho)\} \\ &= \max \left\{ \begin{array}{l} R_c p_n + R_i(1 - p_n) - t_i \rho, \\ R \end{array} \right. \quad \text{choose } s_+ \end{aligned}$$





## SNR-change task thresholds

For a given SNR, the optimal SNR-change task threshold is given by  $m = \frac{2}{\sigma^2} \frac{\mu_2 - \mu_1}{\sigma^2}$  and the corresponding SNR-change task threshold is given by  $\mu(t) = (\mu_2 - \mu_1)H_\theta(t - 0.5) + \mu_1$ .

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**SNR-change task thresholds**



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Author contributions

Concepción V. Wang, Computational analysis, Formal analysis, Visualization, Writing - original draft, Writing - review and editing; Zaira Klacv, Computational analysis, Visualization, Writing - original draft, Writing - review and editing

Author ORCIDs

Concepción V. Wang  <https://orcid.org/0000-0002-3292-9428>  
Zaira Klacv  <https://orcid.org/0000-0002-0192-0493>  
Klára Klacv  <https://orcid.org/0000-0002-1933-3913>  
Zaira Klacv  <https://orcid.org/0000-0002-2933-9418>

Decision letter and Author response

Decision letter <https://doi.org/10.7554/eLife.79824.sa1>  
Author response <https://doi.org/10.7554/eLife.79824.sa2>

Additional files

Supplementary files

 [Supplementary file 1](#)

Data availability

All data and code are available at the following URL: <https://www.nature.com/nature/> [10.1038/s41586-022-03414-9](#).

References

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