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1. I t ct

(A): 2 $2n$ n $n > 1$, z, $z_n \to z$, U z z_n U z U. 1964 [1]

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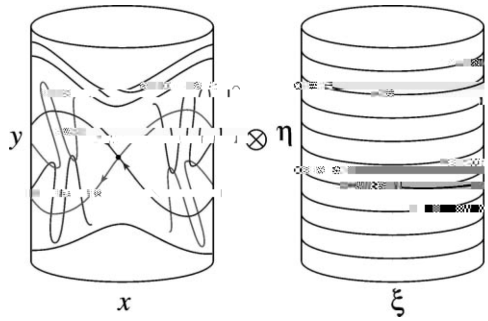


Fig. 1. [illegible] (1) [illegible] $h = 0$ [illegible] (x, y), [illegible] (ξ, η).

[illegible]). [illegible]

[illegible] $T^2 \times R^2$ [illegible]

$$x' = x + y', \qquad \xi' = \xi + \eta', \qquad y' = y - k(1 + h \ [illegible] \ \xi) \ [illegible] \ x, \qquad \eta' = \eta - kh([illegible]$$

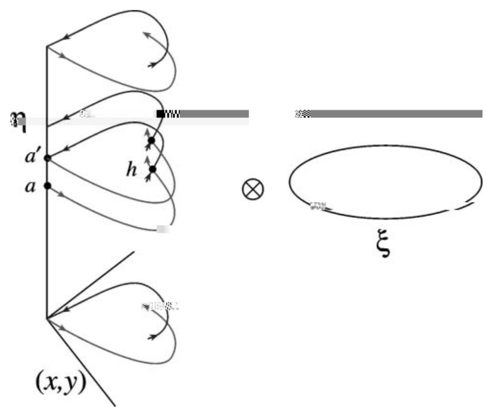


Fig. 2. ... (1) ... $h \neq 0$. ... C ... ξ, ... C_a ... C_a ... $C_{a'}$ ... h.

... a' ... a. ... h

$(x, y) \mapsto (x', y')$,

$$x' = x + \frac{1}{\varepsilon}y', \qquad y' = y + \quad V(x). \tag{4}$$

$n = 1$, $V(x) = k \quad (x)$, [13].

3. A t-- t ab t

[3,4,15], (3)

$$W(x) = \sum_t V(x_t),$$

$\varepsilon = 0$. $V(x_t) = 0$, ( ) V:

$$(V) \equiv \{c : \quad V(c) = 0, \| \quad ^2V(c)\| \geq b > 0\}. \tag{5}$$

A (V) $\varepsilon = 0$. ; $L_2 = 0$, y' $\mathrm{B} = (\mathrm{R}^n)^{\mathrm{Z}}$ $X \in \mathrm{B}$, X

$$A_t(X) \equiv -T_2(x_{t-1}, x_t) - T_1(x_t, x_{t-}$$

4. C[illegible] t[illegible]

[illegible]

that C is not identically zero. Then given any $a < b$, *there is a nonzero measure of initial states* (ξ_0, ξ_1) *and a sequence* $c_t \in .(V)_+ \cup .(V)_-$ *such that the solution of* (14) *has momenta,* $\eta_t = T_2(\xi_{t-1}, \xi_t)$ *satisfying* $\eta_0 < a$ *and* $\eta_T > b$ *for some time T.*

P ... $c_- \in .(V)_-$ ... $c_+ \in .(V)_+$ ... ξ_t, ... $x_t = c_\pm$ ... $(C(\xi_t)) = \pm 1$. ... (14) ...

$$\tilde{L}(\xi, \xi') = T(\xi, \xi') + W(\xi) + \tilde{C}(\xi),$$

... $\tilde{C}$ ...

$$\tilde{C} = V(c_\pm(\xi)) \quad C(\xi) \geq 0.$$

... $\tilde{C}(\xi + 2\pi) - \tilde{C}(\xi) > 0$, ... $F = \tilde{L}(\xi + 2\pi, \xi' + 2\pi) - \tilde{L}(\xi, \xi')$ ... 5, ... □

4.2. Standard example

$$L(x, x', \xi, \xi') = \tfrac{1}{2}\varepsilon(x' - x)^2 + \tfrac{1}{2}(\xi' - \xi)^2 + k \;\; x(1 + h \;\; \xi), \tag{15}$$

... $k > 0$ ... $h > 0$ ... (1), ... (15) ... h

$\xi_t \to \pi$, … $\eta^* = 2\pi m$. … $(0, 2\pi m)$ … $(\pi, 2\pi m)$, … (16). … $\eta^* = (2m+1)\pi$, … $(0, \eta^*) \mapsto (\pi, \eta^*)$. … (16) … (…). … (16) …

… (16), … g^* … g … g^* … g. … (16) … 2π … $\mathrm{T}^2 = \{(\xi, \eta) : 0 < \xi, \eta < 2\pi\}$. …

$$\eta^* = (\eta_{t+1} - \eta_t)^* = \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} (U'(\xi_t)).$$

…

$$\langle \eta^* \rangle = \frac{1}{4\pi^2} \int_{\mathrm{T}^2} U'(\xi) = \frac{F}{2\pi}.$$

… (16) … .012

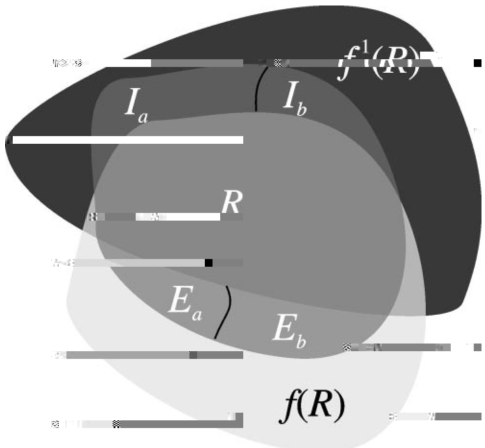


Fig. 4. … R.

…

$$E = \{z \in R : f(z) \notin R\} = R \setminus f^{-1}(R).$$

… f … μ, … R …:

$$\mu(E)=\mu(R \setminus f^{-1}(R))=\mu(R)-\mu(R\cap f^{-1}(R))=\mu(R)-\mu(f(R)\cap R)=\mu(R\setminus f(R))=\mu(I). \qquad (17)$$

… S^t … t … I … *stays* … R … t. …

$$S^0 = I, \qquad S^t = f(S^{t-1}) \cap R = f(S^{t-1} \setminus E).$$

… S^t … $f^{-j}(S^t)$ … R … $j = 0, \dots, t$, … $(t+1)^{st}$, … S^t … R, … $\sum_{t=0}^{\infty} \mu(S^t) < \mu(R)$. … $\mu(S^t) \to 0$ … $t \to \infty$. … I … R … E. …

E_b

$$\mu(p(I_a) \cap E_b) = \mu(p(I_a)) - \mu(p(I_a) \cap E_a) \geq \mu(I_a) - \mu(E_a). \qquad \square$$

A f_t R, t

$$I_t = R \setminus f_t(R), \qquad E_t = R \setminus f_t^{-1}(R).$$

f_t (17), R k t S_k^t.

$$S_k^k = I_{k-1}, \qquad S_k^{t+1} = f_t(S_k^t \setminus E_t).$$

A S_k^t t $k \leq t$ R

$$\sum_{k=-\infty}^{t} \mu(S_k^t) < \mu(R) \tag{18}$$

t.

Lemma 4. *Let f_t be a sequence of measure-preserving homeomorphisms, and R a measurable set with incoming sets I_t and exit sets* t 1012(

5.2. Maps of the cylinder

… F, … C, … C … f(C)

$$\mathrm{F} = {}_{C}\; y',\, x' - y,\, x.$$

… F … C.
… C, … T … B. … U ⊂ T …

$$U = \{z \in T : f^{-1}(z) \in B\}.$$

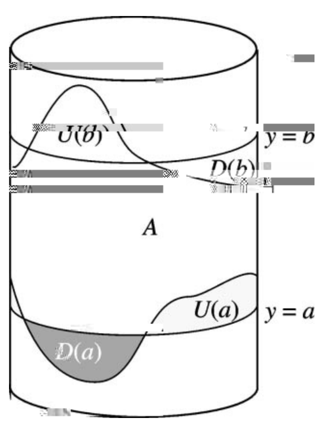
… D ⊂ B …

$$D = \{z \in B : f^{-1}(z) \in T\}.$$

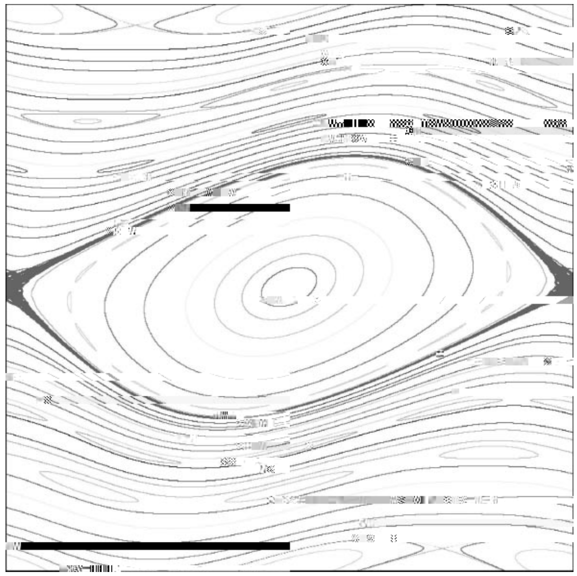
… $\mu(U) - \mu(D) = \mathrm{F}$ …
… 4 …

C… a… 5. *Suppose that f_t is a sequence of area and end-preserving homeomorphisms of the cylinder, and that the net flux $\mathrm{F}_t \geq \delta > 0$. Let A denote the annulus bounded by the circles $\{y = a\}$ and $\{y = b\}$ where $a < b$. Then, there is a set of positive measure of orbits that cross the annulus.*

P… … $U_t(a)$ … $D_t(a)$ … $\{y = a\}$ … f_t, … b, … 5. … A … $I_t = U_t(a) \cup D_t(b)$, … $E_t = f_t^{-1}(D_t(a)) \cup f_t^{-1}(U_t(b))$. … $\mu(U_t(b)) \geq \delta > 0$, … 4, … a … b … $f_t^{-1}(U_t(b))$. □



… 5. …



. 6. … $k = 0.5$. …

5.3. Standard map with net flux

… (4) … $(0, 2\pi) \times \mathrm{R}$. … $\mathrm{F} = V(2\pi) - V(0)$. … V … :

$$x' = x + y', \qquad y' = y - k \ldots (x) + \frac{1}{2\pi}\mathrm{F}.$$

… $\mathrm{F} = 0$, … $k < k_{cr} \approx 0.971635406$ … $k = 0.5$ … . 6. … 5, … $\mathrm{F} \neq 0$ … $y = 0$ … $y = 2\pi$. … m, $f(x, y + 2\pi m) = f(x, y) + 2\pi(m, m)$. … . 7, … F … F … $y = 0$ …

$$x = \ldots \frac{\mathrm{F}}{2\pi k}$$

… $\mathrm{F} = 2\pi k$.

6. P… t c… t

… (12) … ε. … $z_{t-1} = (x_{t-1}, x_t, \xi_{t-1}, \xi_t)$ …

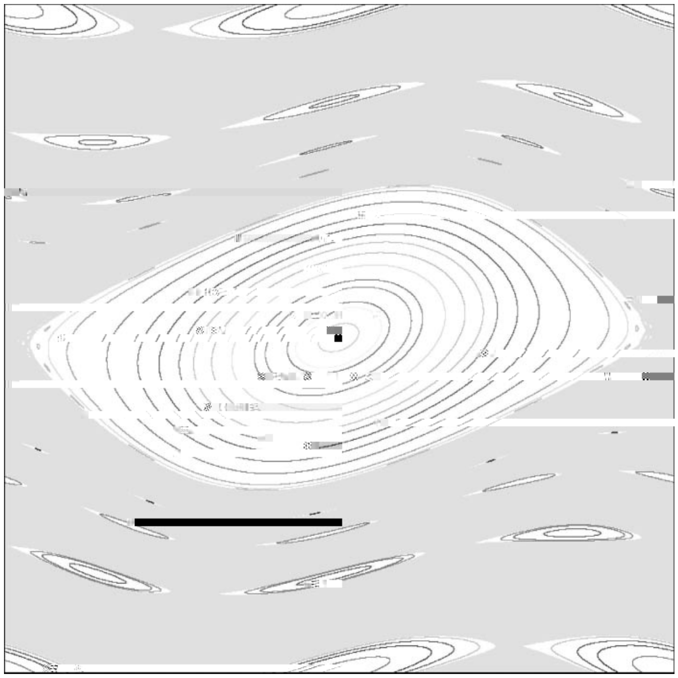


Fig. 7. ... $k = 0.5$, ... $\mathcal{F} = 4\pi^2/1000$. ... (...).

$$z_t = \phi_\varepsilon(z_{t-1}) = \begin{pmatrix} x_t \\ -x_{t-1} + 2x_t + \dfrac{1}{\varepsilon} \; V(x_t)[1 + C(\xi_t)] \\ \xi_t \\ -\xi_{t-1} + 2\xi_t + \; W(\xi_t) + V(x_t) \; C(\xi_t) \end{pmatrix}. \tag{19}$$

... $\varepsilon = 0$, ... $x_t = c_t \in$ ...(V) ... ϕ_0. ... $\varepsilon > 0$, ... x- ...

L... a 6. *Suppose that ϕ_ε, given by* (19), *is a C^2 map of T^4, such that $1 + C(\xi) \geq \tau > 0$. Then, for any sequence $\{c_0, c_1, \dots\}$ with $c_t \in$ ...$(V) \cap \mathrm{A}$, any initial condition (ξ_0, ξ_1), and any $\delta > 0$, there exists an orbit $z_t = (x_t, x_{t+1}, \xi_t, \xi_{t+1})$, $t \geq 0$ of ϕ_ε such that*

$$|x_t - c_t| \leq \delta \quad \ldots \quad t \geq 0,$$

provided

$$0 \leq \varepsilon < \varepsilon_0 = \frac{\sigma\tau}{(4\delta + a)}, \tag{20}$$

where $\sigma(\delta, b) \equiv$ ... $_{t\geq 0}| \; V(c_t \pm \delta)|$.

a $1 + C(\xi) \geq \tau$
$(1/\varepsilon)\ V(c^{+}_{t+1})\tau > (4\delta + a)$ (ε) $\sigma\tau/\varepsilon > 4\delta + a$ (20).
W_t B_{t+1} W_{t+1}.
S W_0 $S = \mathbb{R}^2 \times (\xi_0, \xi_1) \cap W_0$.
z_0 S T S
U_t W_t $t \geq 1$. B
S
W_0, T B
T B W_t
S T B.
S W_t . □

2, (ξ_0, ξ_1) $\{c_t\} \in$ (V)
$Z_t = (c_t, c_{t+1}, \xi_t, \xi_{t+1})$
ε $\{Z_t\}$ $\{z_t\}$ ϕ
T ε

T 7. *Suppose that ϕ_ε satisfies the hypotheses of Lemma* 6. *Let $Z_t = (c_t, c_{t+1}, \xi_t, \xi_{t+1})$ be an orbit of ϕ_0 with $c_t \in$ (V). Then for any $T \geq 0$ and $\gamma > 0$, there is a $\delta > 0$ such that for all $\varepsilon < \varepsilon_0(\delta)$ in* (20.5716 0 0 7.5716 439. 1

$|\lambda_t^i| \le r^t$, $r > 1$ $r^2 - wr - 1 = 0$, $w =$ $_x(2 + |$ $W(x)|)$.

$$|\lambda_t| \le \tfrac{1}{2} M \delta^2 r^{2t}.$$

$W = 0$, λ_t t^2 T γ δ $|\lambda_t| \le \gamma$ $t \le T$. □

R **a** ξ_t $C(\xi)$,

6.1. Standard example, continued

7 (15), $V(x) = k$ x, $\delta < \pi/2$, $\sigma = k$ δ. $C(\xi) = h$ ξ, $\tau = 1 - h$, $h < 1$. $c_t \in \{0, \pi\}$, (x, y) $a = 2\pi$. 6

$$\varepsilon \le \varepsilon_0 = \frac{k(1-h)\ \ \delta}{4\delta + 2\pi}.$$

7 $M = kh$ $W = 1$

$h < 1$ $\varepsilon \leq \varepsilon_0$. ε 9.

7. C c

[7] [6,9,20] [19]

[16] (x, y) [17].

Ac t

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